Separation of Proof and Program

The Trellys Project

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MVD 2011
Introduction

- The design of a core language of a new dependently typed programming language Trellys.
  - Separation of Proof and Program ($\text{Sep}^3$).

- The logical fragment.
  - Equality, explicit conversions, a new termination predicate, case splitting on programs, and induction using a primitive ordering.

- The programmatic fragment.
  - General recursion, explicit conversions, and case splitting on programs.
Sep$^3$ is a call-by-value language that consists of two fragments, a logical fragment and a programmatic fragment.

- The language syntactically separates the logical and programmatic fragments.
- The logical fragment is a predicative higher-order logic.
- While the programmatic fragment contains general recursion and type:type.

The two fragments are separate, but they are linked.

- Proofs can “talk” about programs, but are not allowed to run them.
- Programs can contain proofs.
The Logical Fragment

- A predicative higher order logic.
  - The logic is weakly constructive. What this means is that there is only one predicate that forces the logic to be non-constructive.

- The logical fragment is compile time only.
  - That is all proofs are erased during compile time.

- The logic has the following as primitives.
  - Disjunction, existential types, absurdity, higher-order predicative quantification, implication, propositional equality, explicit conversions, induction, and a new termination predicate.
Equality and Conversion

- The logic of Sep\(^3\) has a primitive notion of propositional equality.
  - This equality is a typed equality and expresses when two programs are equivalent.
  - Intro. form: \(\Gamma \vdash join\ n\ m : t_1 = t_2\).
  - Use: Suppose \(|t|\) is a function that erases all the proofs from the program \(t\) then we if \(|t_1| \Rightarrow^n t'\) and \(|t_2| \Rightarrow^m t'\) and \(t_1\) and \(t_2\) are typeable then we may conclude that \(t_1\) and \(t_2\) are equivalent with the proof \(join\).

- Explicit conversion adds the ability to make use of equalities between programs.
  - Elim. form: \(\Gamma \vdash conv\ p\ by\ eqpf\ at\ hole.p : [t_2/hole]P\)
  - Use: If we know \(p\) is a proof of \([p_1/hole]P\), and we can prove \(t_1 = t_2\) then we can replace \(t_1\) with \(t_2\) in \([p_1/hole]P\) and obtain a new proof of \([p_2/hole]P\).
Termination

- The logic contains a new predicate called the termination predicate.
  - The termination predicate internalizes the notion of termination.
  - Predicate form: $t!$.
  - Explanation: For some program $t$ if we can prove $t$ normalizes then we may conclude $t!$.
- We not only need to show that $t$ normalizes, but that the normal form of $t$ can be judged a value.
  - Intro. form: $\Gamma \vdash \text{val} \alpha x \; t : t!$.
  - Use: If we can judge $t$ a value, denoted $\Gamma \vdash \text{val} \; t$, then we may conclude with the proof $\Gamma \vdash \text{val} \alpha x \; t : t!$ which states that $t$ has a value.
- Forms that may be judged values:
  - $\lambda$-abstractions, $\text{Type}$, recursors, data type constructors whose arguments are values, and variables marked as values.
Termination: An Example

Example

Suppose $t$ is a program and $v$ is $t$'s value. Then

\[
\begin{align*}
|t| & \downarrow |v| \\
\Gamma & \vdash t : t' \\
\Gamma & \vdash v : t' \\
\Gamma & \vdash \text{val } v
\end{align*}
\]

\[
\begin{align*}
\Gamma & \vdash \text{join } m \ n : t = v \\
\Gamma & \vdash \text{valax } v : [v/hole](\text{hole!})
\end{align*}
\]

\[
\Gamma \vdash \text{conv}(\text{valax } v) \text{ by (join } m \ n) \text{ at hole. (valax hole)} : t!
\]
How can we use the termination predicate?

- If p is a proof of t! for some program t then t can be used as a value.
  - Form: $\Gamma \vdash \text{val tcast t by } p$.
- $tcase$ provides the ability to case split on the termination behavior of programs.
  - DISCLAIMER: t! is not constructive.
  - Form: $\Gamma \vdash tcase t[\alpha] \text{ of abort } \rightarrow p_1 \mid ! \rightarrow p_2 : P$.
  - Use: For some program t if $p_1$ is a proof of some predicate P assuming t! and $p_2$ is a proof of P assuming t diverges then $tcase t[\alpha] \text{ of abort } \rightarrow p_1 \mid ! \rightarrow p_2$ is a proof of P.
Induction

Sep\(^3\) has a primitive notion of structural course-of-values induction.

Form: \(\Gamma \vdash ind\ f\ x : t_1,\ u.p : \forall x : t_1.\forall u : x!.P\).

Use: If \(p\) is a proof of some predicate \(P\) assuming
\(\forall y : t_2.\forall u : y < x.[y/x]P\) holds then we can prove \(P\) for any
program \(x\) of type \(t_1\).
The Programmatic Fragment

- The programmatic fragment has a collapsed syntax. Terms and types are all generated by the same syntactic category.

- Type:Type

- General recursion.
  - Form: $\Gamma \vdash rec\ f\ x : t_1.t : \Pi x : t_1.t_2$.

- Explicit conversions.

- Data types and case splitting on programs.
  - Intro. Form: data C t where \{C_1 : t_1, \ldots, C_n : t_n\}.
  - Elm. Form: case t [eq_pf] of C_1 t_1 \ldots t_n | \cdots | C_k t'_1 \ldots t'_2 | done.
Concluding remarks

- Future work.
  - Complete the meta-theory.
  - Design and implement the surface language.

- Thank you all for listening.